CHAPTER 2

2/1 $v_o = 12$ mi/h = 17.6 ft/s and $x_o = 0.0$ ft.

Interval $0 \le t \le 5$ s.

$\frac{dv}{dt} = a = t$ ft/s^2. By integration, $v = \frac{t^2}{2} + 17.6$ ft/s.

$\frac{dx}{dt} = v$ and $x = \frac{t^3}{6} + 17.6\,t + 0.0$ ft.

When t = 5 s, v(5) = 30.1 ft/s and x(5) = 108.8 ft.

Interval $5 \le t \le 15$ s or $0 \le (t - 5) \le 10$

$\frac{dv}{dt} = a = 5$ ft/s^2 and $v = 5(t - 5) + 30.1$ ft/s.

$\frac{dx}{dt} = v$. Therefore, $x = 5\,\frac{(t - 5)^2}{2} + 30.1(t - 5) + 108.8$ ft.

When t = 15 s, v(15) = 80.1 ft/s and x(15) = 659.8 ft.

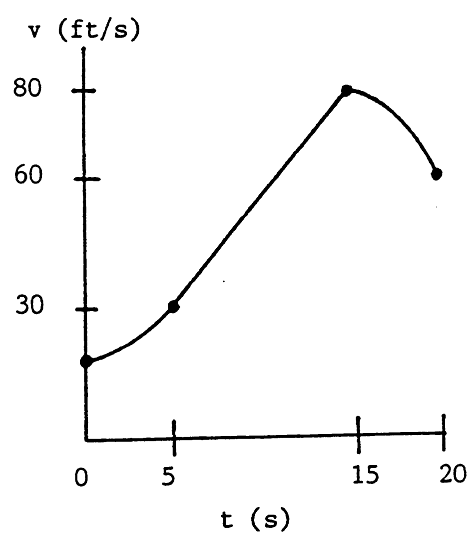
Interval $15 \le t \le 20$ s or $0 \le (t - 15) \le 5$

$\frac{dv}{dt} = a = -\frac{8}{5}(t - 15)$ ft/s^2 and $v = -\frac{8}{5}\,\frac{(t - 15)^2}{2} + 80.1$ ft/s.

$\frac{dx}{dt} = v$. Consequently, $x = -\frac{8}{5}\,\frac{(t - 15)^3}{6} + 80.1(t - 15) + 659.8$ ft.

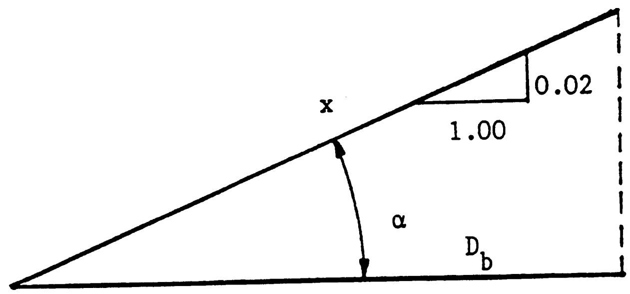
When t = 20 s, v(20) = 60.1 ft/s and x(20) = 1027.0 ft. Answer

The relationship between speed and time is plotted below.



Note that the shape of the v-t curve can be inferred directly from the shape of the a-t diagram. At t = 0, the slope of the v-t diagram is zero since a = 0. The slope increases in a linear fashion until t = 5 s. Between t = 5 s and t = 15 s, the slope remains constant. At t = 15 s, it abruptly changes to zero, and then it decreases linearly.

2/2



From the above diagram $\tan\alpha = 0.02$ and $\alpha = 1.15°$.

Also $D_b = x\cos\alpha \simeq x$.

Eq. 2.2.6 gives $x = \dfrac{v^2 - v_0^2}{(2)(8)}$ and Eq. 2.2.13 yields $D_b = x = -\dfrac{v^2 - v_0^2}{2g(f + 0.02)}$

Therefore $16 = 2g(f + 0.02) = 64.4\,(f + 0.02)$.

Solving for the coefficient of friction, $f = 0.23$.

This value suggests a wet pavement.

2/3

Assuming the case of constant acceleration,

$v = at + v_0$ and $(v^2 - v_0^2) = 2a(x - x_0)$ [Eqs. 2.2.4 & 2.2.6]

The movement from the ground floor to the restaurant level involved:

Total distance = 140 ft.

Time to reach cruising velocity when $a = 5\ \text{ft/s}^2 = \frac{20}{5} = 4$ s.

Time to stop from cruising velocity when $d = 4\ \text{ft/s}^2 = \frac{20}{4} = 5$ s.

Acceleration distance = $20^2/[2(5)] = 40$ ft.

Deceleration distance = $20^2/[2(4)] = 50$ ft.

Cruising distance = 140 − 40 − 50 = 50 ft.

Cruising time at maximum cruising speed = 50/20 = 2.5 s.

During the movement from the restaurant level to the observation deck the elevator did not reach cruising velocity. The total distance of 20 ft consisted of accelerating (x_a) and decelerating (x_d) distances, i.e.,

$$x_a + x_d = 20 \text{ ft.}$$

2/3 (cont.)

Hence, $$\frac{v^2}{2(5)} + \frac{v^2}{2(4)} = 20 \text{ ft.}$$

Consequently, the highest speedreached was v = 9.4 ft/s. In addition,

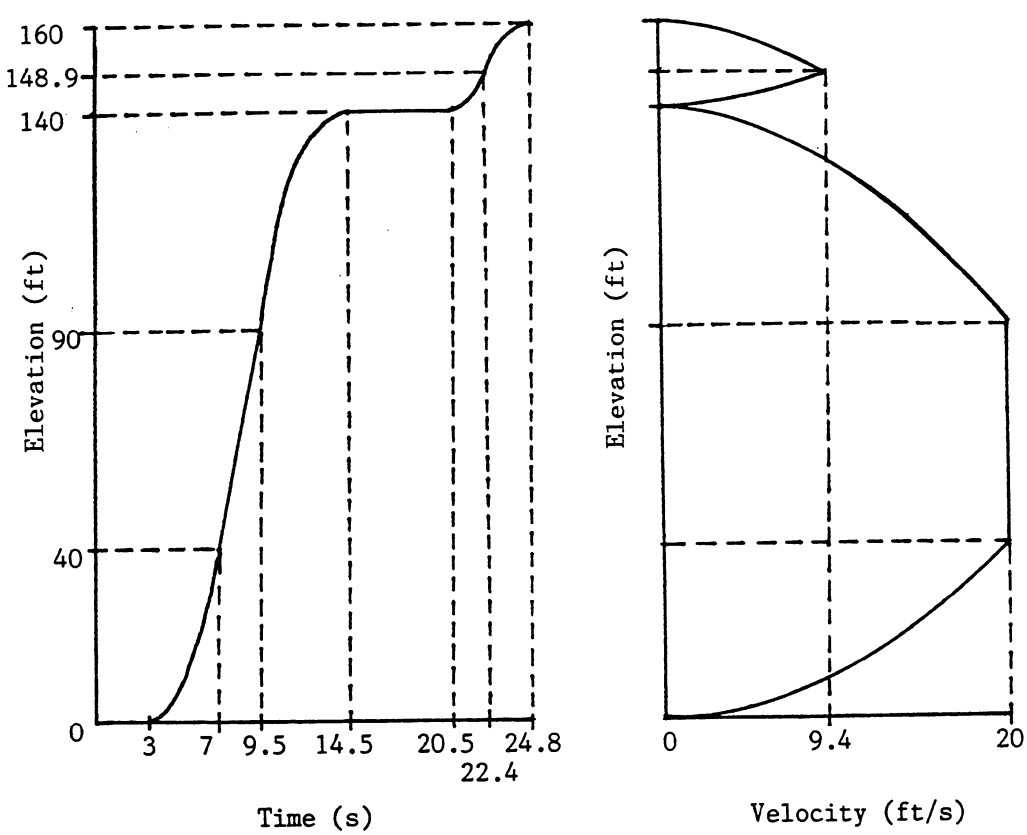
Acceleration distance ≃ 8.9 ft.

Deceleration distance ≃ 11.1 ft.

Acceleration time ≃ 1.9 s.

Deceleration time ≃ 2.4 s.

The required diagrams are drawn below.



2/4 $A = 100 \text{ ft}^2$ $W = 40{,}000 \text{ lb}$ $\alpha = 100 \text{ lb/ft}^2$ $\beta = 3.33 \text{ lb/ft}^2\text{-s}$

a) $$F = (\Delta P)A = (W/g)a$$

Solve for acceleration in terms of pressure difference ΔP:

$$a = \frac{g}{W} A\ (\Delta P) = \frac{32.2}{40{,}000}(100)(\Delta P) = 0.0805(\Delta P)$$

Also, $v = \int a\, dt$ and $x = \int v\, dt$.

For simplicity, set $t_0 = 0$ and $x_0 = 0$.

2/4 (cont.)

Acceleration phase ($0 \leq t \leq t_1$):

$$a = 0.0805(100 - 3.33t) = 8.05 - 0.268t \quad ft/s^2$$

$$v = 8.05t - 0.268(t^2/2) + v_0 = 8.05t - 0.134t^2 \quad ft/s \qquad (Eq.1)$$

$$x = 8.05(t^2/2) - 0.134(t^3/3) + x_0 \quad ft.$$

According to the given a–t diagram, $a = 0$ when $t = t_1$. Consequently, $t_1 = (8.05)/(0.268) = 30$ s. At this instant, cruising velocity is attained:

$$v_{cruise} = 8.05(30) - 0.134(30)^2 = 120.9 \quad ft/s.$$

The distance traveled during the acceleration phase is $x_a = 2416.5$ ft.

Deceleration phase ($t_2 \leq t \leq t_3$):

$$a = 0.0805(-3.33)(t - t_2) \qquad \text{where } t_2 \text{ depends on station spacing.}$$

$$v = -0.268\frac{(t - t_2)^2}{2} + v_{cruise} = 120.9 - 0.134(t - t_2)^2 \qquad (Eq.2)$$

$$(x - x_2) = 120.9(t - t_2) - 0.134\frac{(t - t_2)^2}{3}$$

The deceleration time may be computed via Eq. 2 or by symmetry with the acceleration phase to be $(t_3 - t_2) = 30$ s. By similar reasoning, the deceleration distance x_d equals the acceleration distance x_a , that is 2416.5 ft.

Cruising phase ($t_1 \leq t \leq t_2$):

The total cruising distance equals the station spacing (1 mi = 5280 ft) minus $(x_a + x_b)$, or $x_{cruise} = 447$ ft. The required equations for the cruising phase are:

$$a = 0 \; ft/s^2 \qquad v = 120.9 \; ft/s \quad \text{and} \quad x = 120.9(t - 30) + 2416.5 \quad ft.$$

b) The v–t diagram for the entire movement is shown below: